

# The Equation of State of Neutron Stars: Neutron-star masses, radii and internal composition.

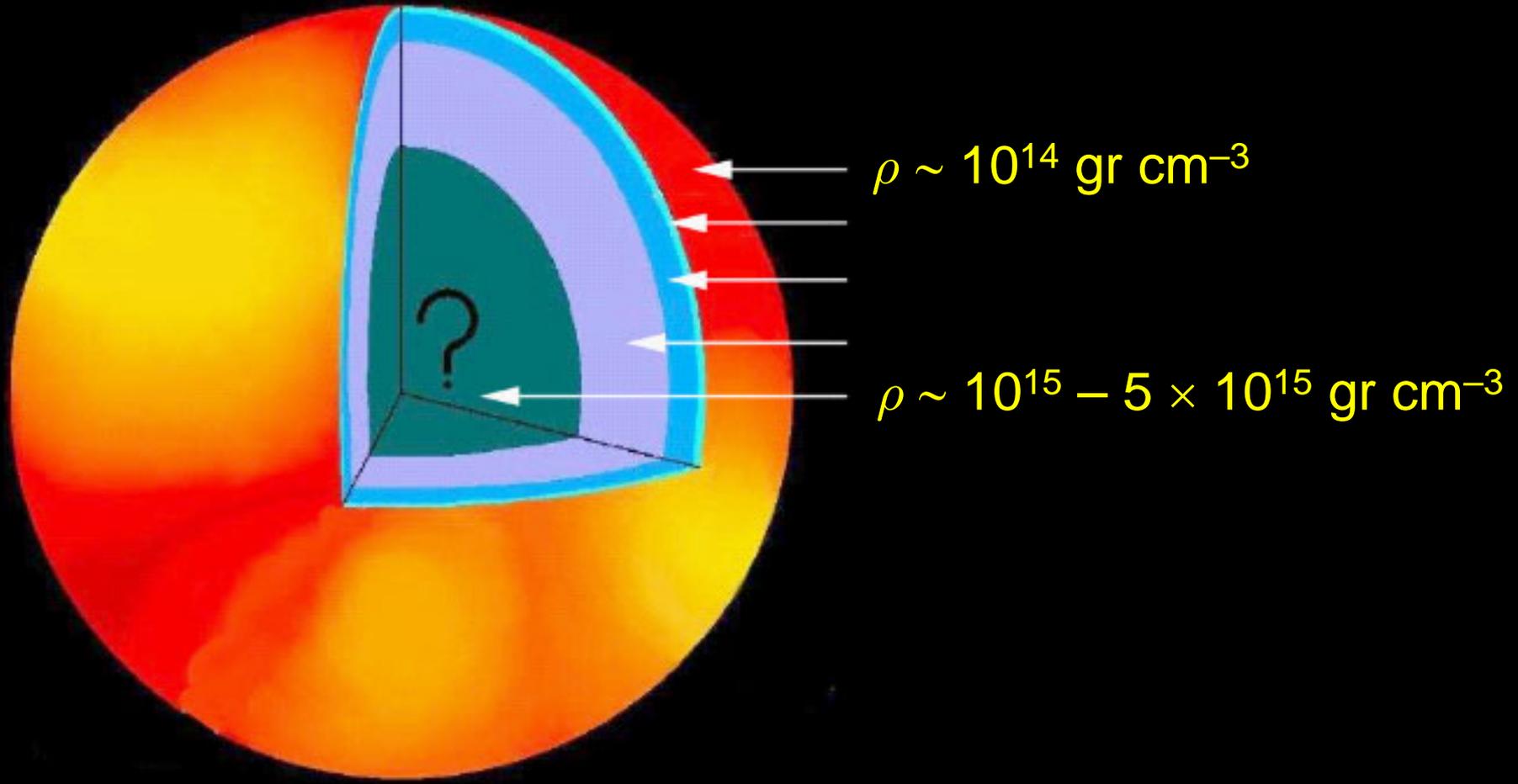
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The Netherlands*

*Didier Barret (CESR), Frits Paerels (Columbia), Tod Strohmayer (NASA/GSFC), Phil Uttley (Southampton), Jon Miller (Michigan), Joern Wilms (Bamberg) ...*

*... and many others ...*

# Neutron-star structure



*Figure courtesy of D. Page*

# Equation of State of nuclear matter

1. Use hydrostatic equilibrium and mass conservation in GR:

$$\frac{dP}{dr} = -\frac{G\rho m}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi Pr^3}{mc^2}\right) \left(1 - \frac{2Gm}{c^2 r}\right)^{-1}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

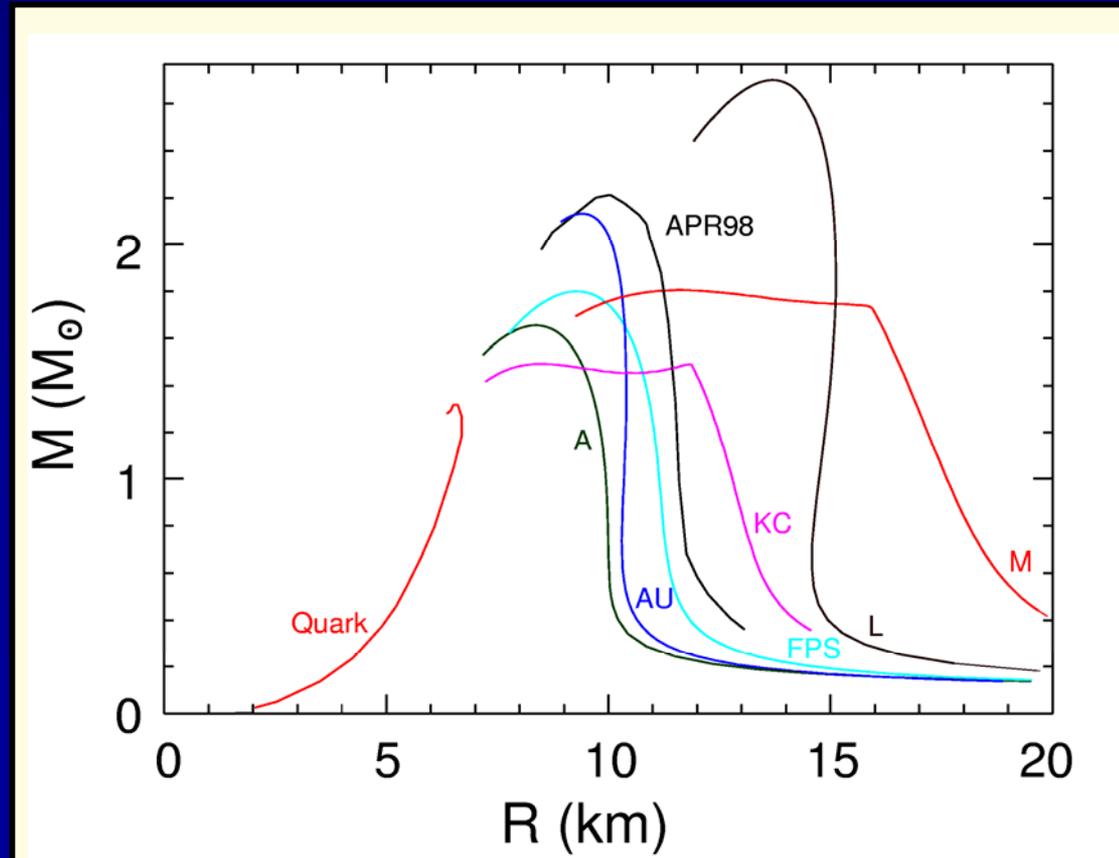
2. Add a prescription for the relation between pressure and density,  $P = P(\rho)$ .

3. Integrate from  $P(r=0) = P_c$  to  $P = 0$ , which defines  $M$  and  $R$ .

For each prescription  $P = P(\rho)$ , this yields a family of solutions as a function of the initial condition,  $P = P_c$ .

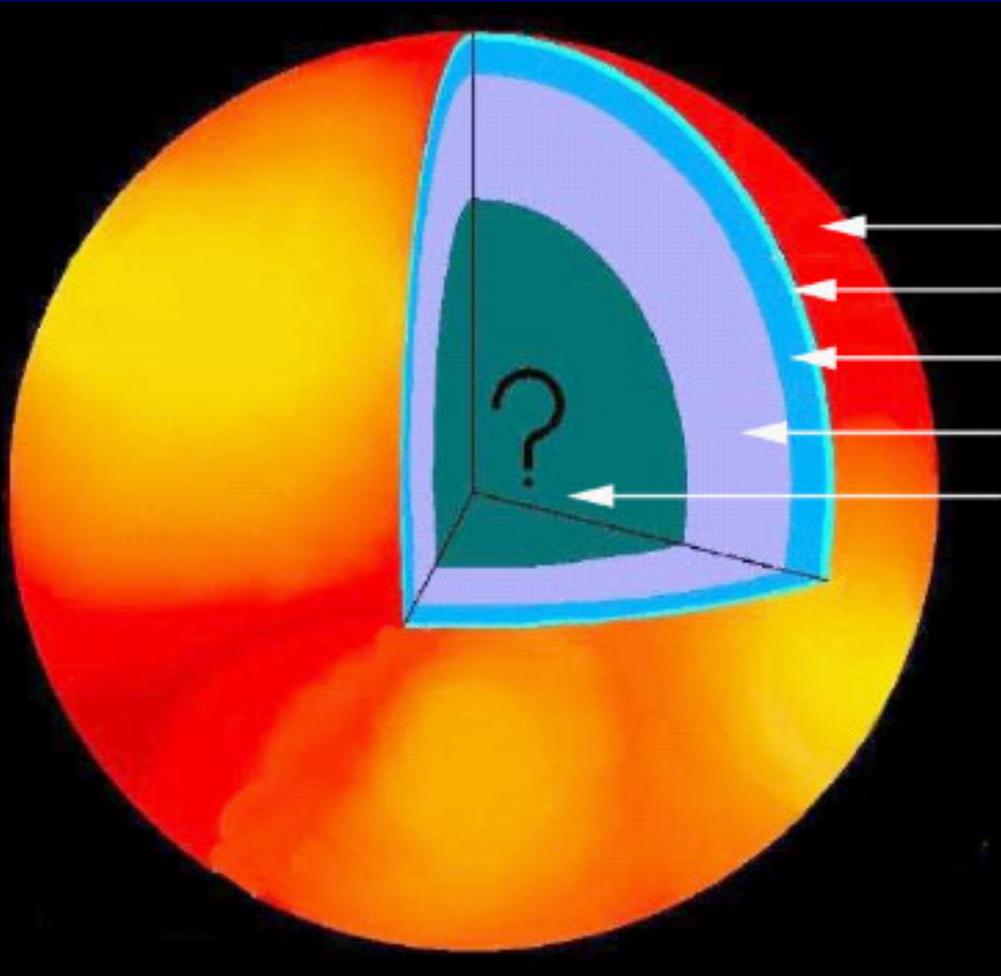
# Equation of State of nuclear matter

The relation between  $P = P(\rho)$ , the so-called equation of state (EOS), is set by the interactions between the particles that constitute the star, and can therefore be mapped into a mass-radius relation,  $M = M(R)$ .



*Cook, Shapiro & Teukolsky;  
Akmal, Pandharipande, Ravenhall;  
Heiselberg*

# Equation of State of nuclear matter



EOS reasonably well-known for the outer parts, but unconstrained for the high-density core.

Uncertainty due to inability to extrapolate our knowledge of normal nuclei (with 50% proton fraction) to the high-density regime of nearly 0% proton fraction.

# Neutron-star EOS: Why?

- QCD (e.g., existence of Bose-Einstein condensates or free quarks at low temperatures); relevant to high-energy and particle physics.
- Dynamics of supernovae explosions.
- NS–NS mergers, which are likely progenitors of short GRBs and sources of strong gravitational waves.
- Stability of neutron stars.
- NS cooling which, compared to observed NS temperatures, provides NS ages.

# Equation of State of nuclear matter.

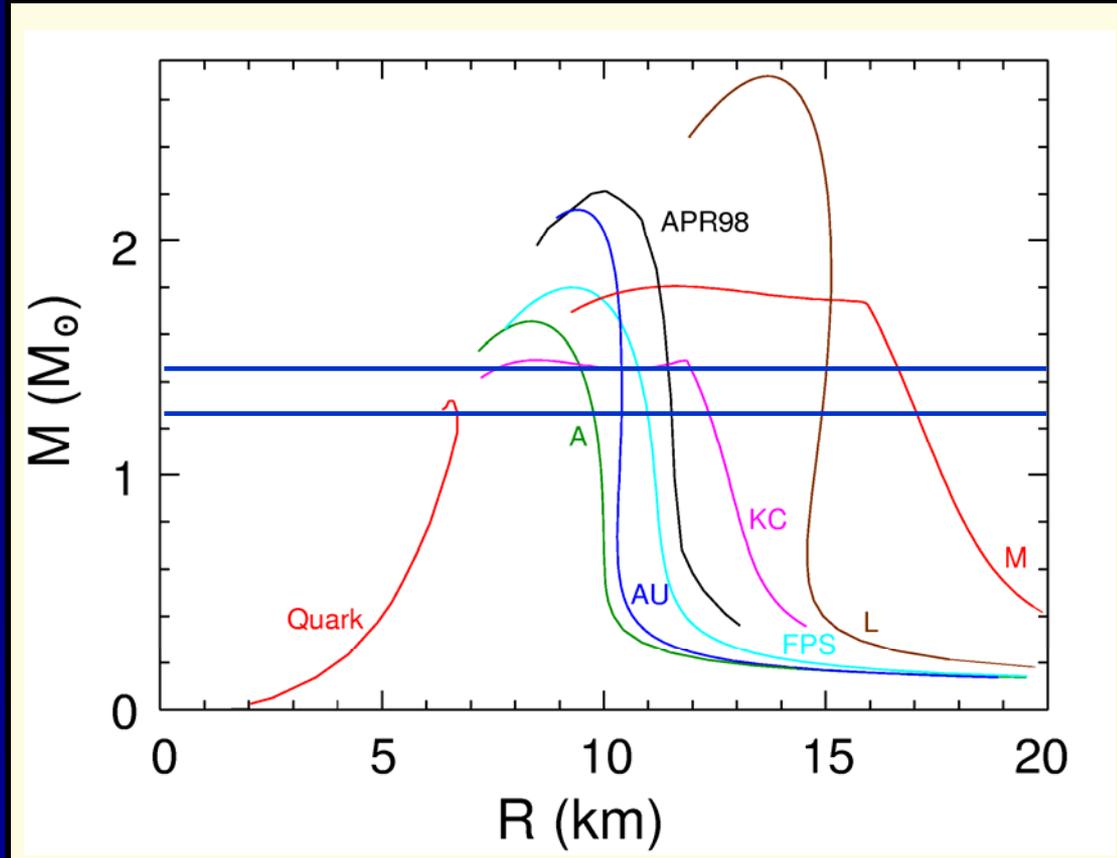
From pulsars in binaries  
 $\langle M \rangle = 1.35 \pm 0.04 M_{\text{sun}}$

Some masses accurate  
down to 0.1% (!)

Mass alone not enough  
to exclude any EOS.

A combination of mass  
and radius required.

... or a massive NS.



# Neutron star EOS measurements and constraints

## Time-resolved spectroscopy and photometry:

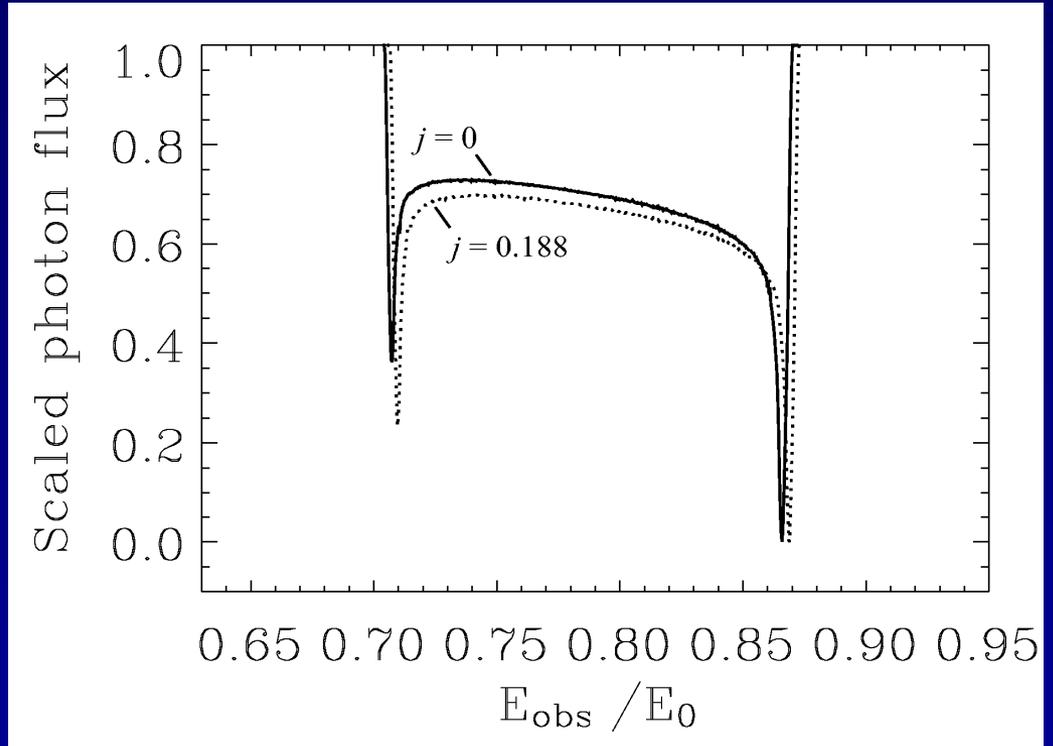
- ▶ Redshifted photospheric lines  $\rightarrow M/R$  (potentially  $M/R^2$ )
- ▶ Profile of photospheric lines  $\rightarrow M/R$
- ▶ Pulsations waveform  $\rightarrow M/R$
- ▶ Quasi-periodic oscillations  $\rightarrow R(M)$  (from disc)
- ▶ Fe emission (disc) lines  $\rightarrow R(M)$  (from disc)
- ▶ Frequency-resolved time-delay spectrum  $\rightarrow R$  (from disc)



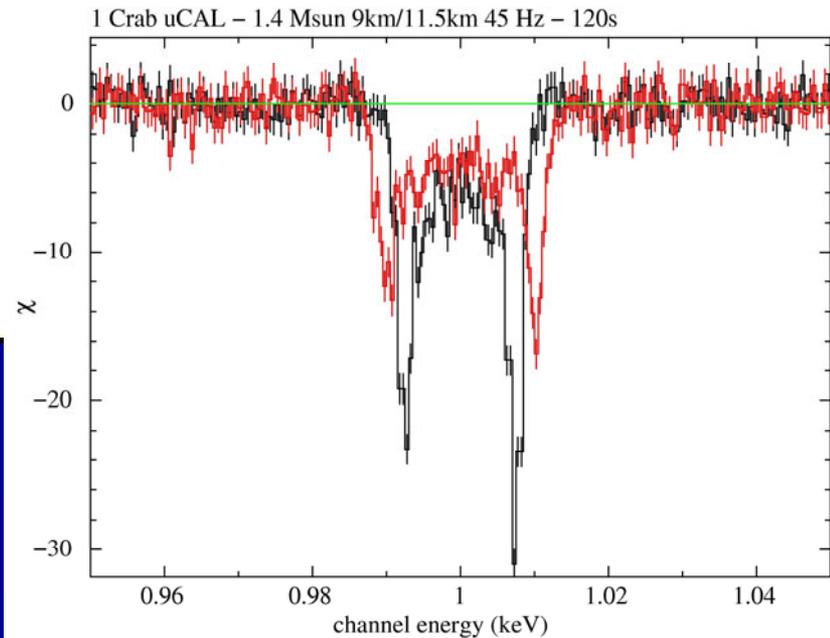
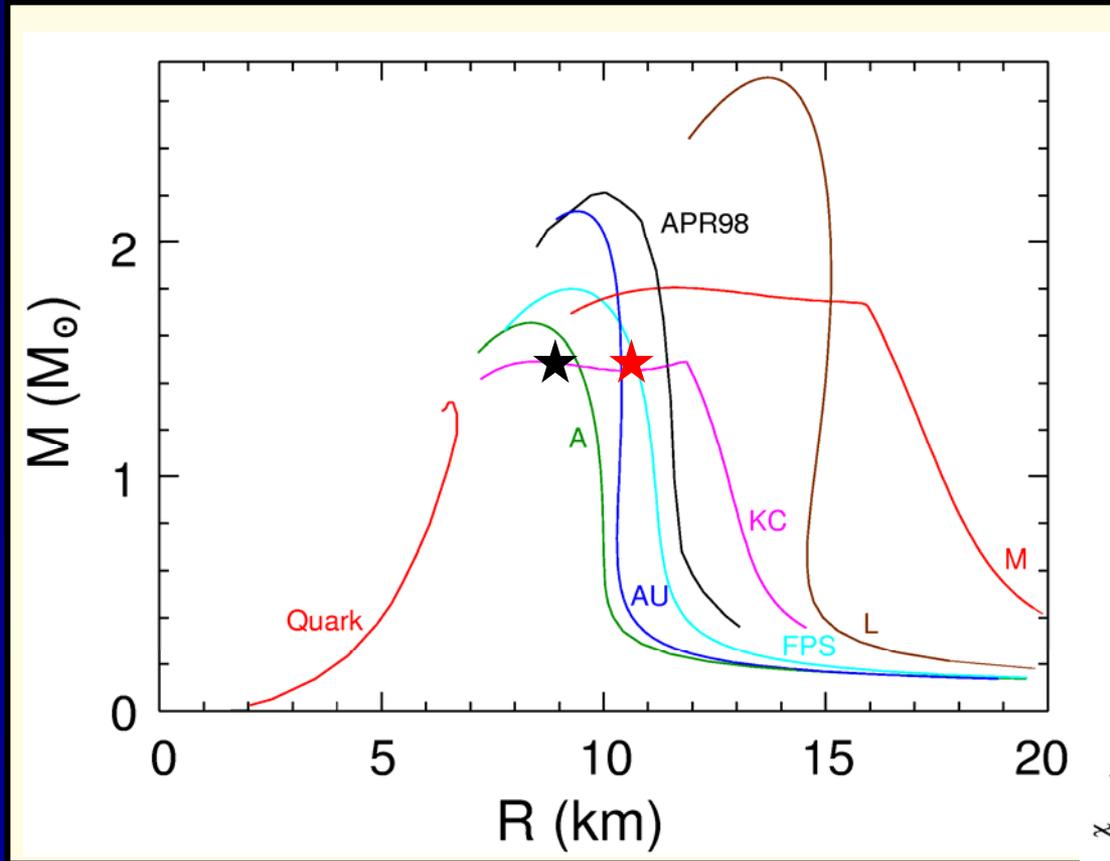
# Spectral line profile

Line profile set by:

- longitudinal and transverse Doppler shifts
- special relativistic beaming
- gravitational redshift,
- light-bending
- frame-dragging



# Simulated spectral line profile



*Line profile calculations courtesy of  
S. Bhattacharyya*

# XMS for bright sources

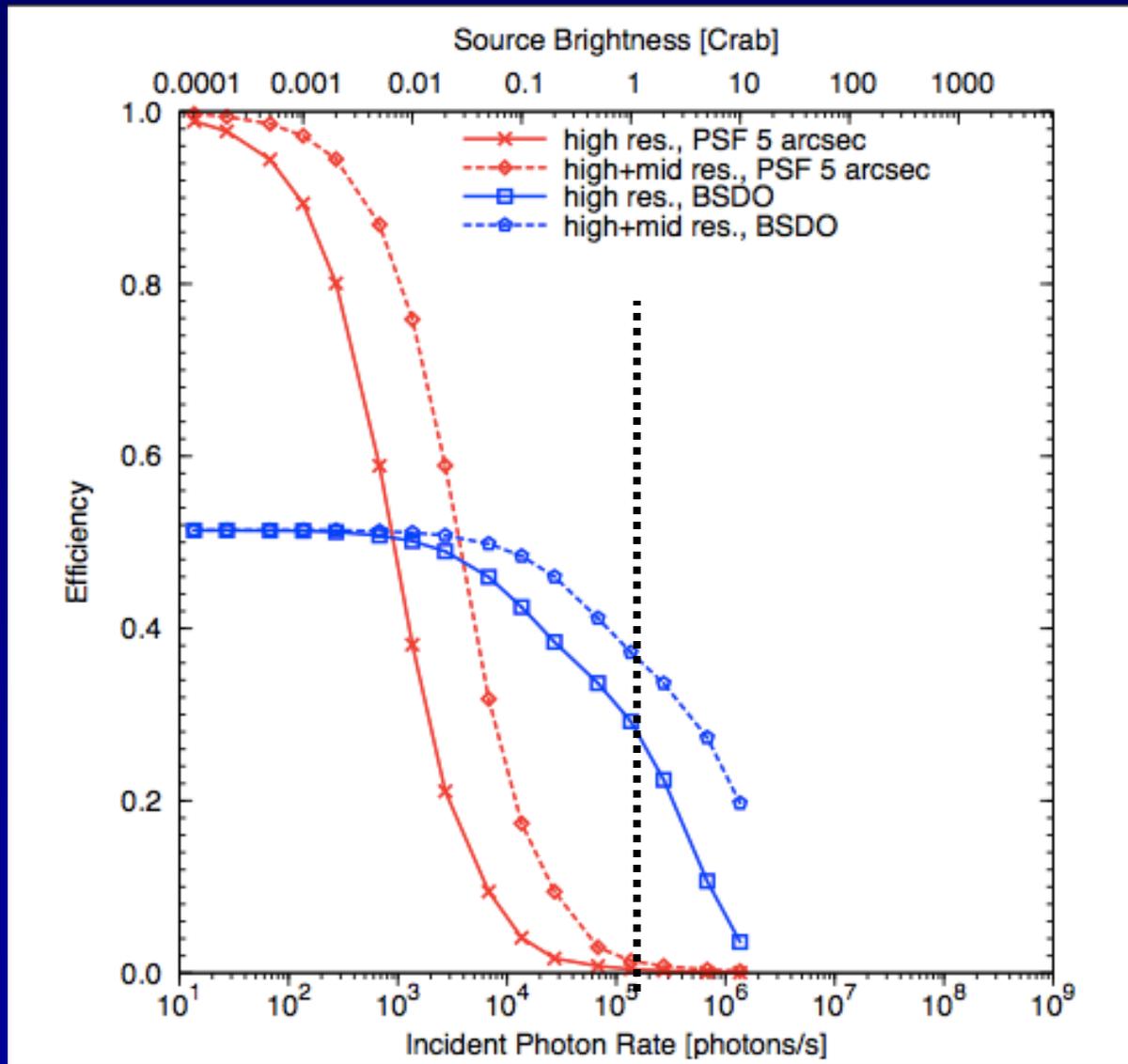


Figure from J. Wilms et al.

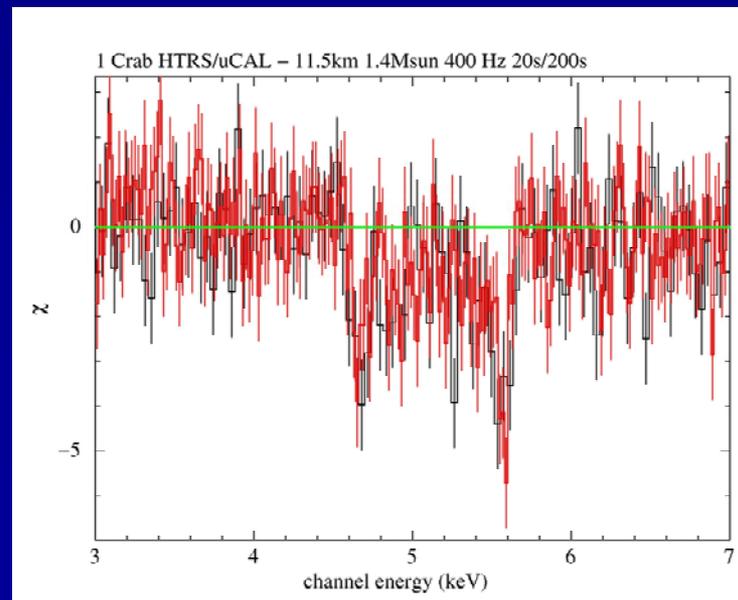
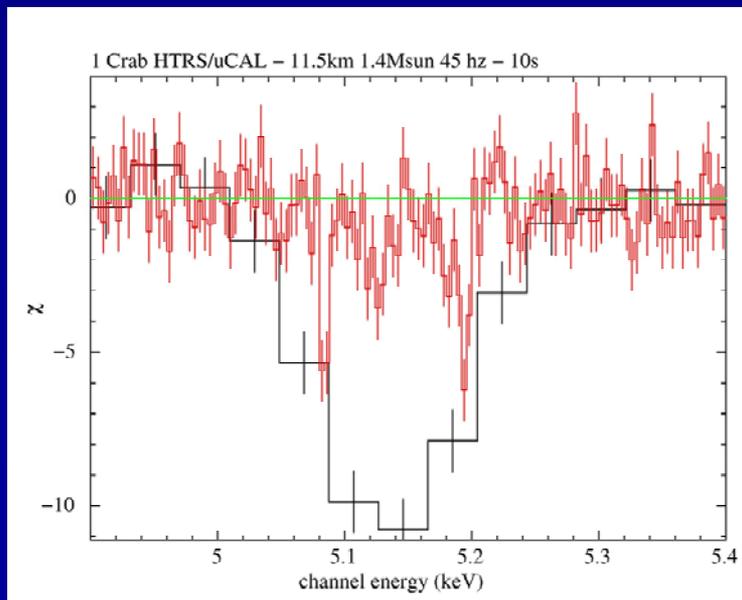
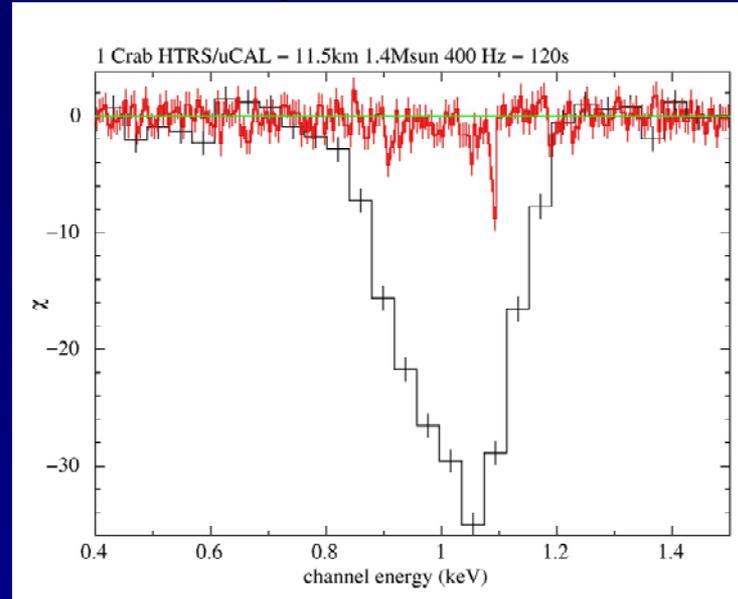
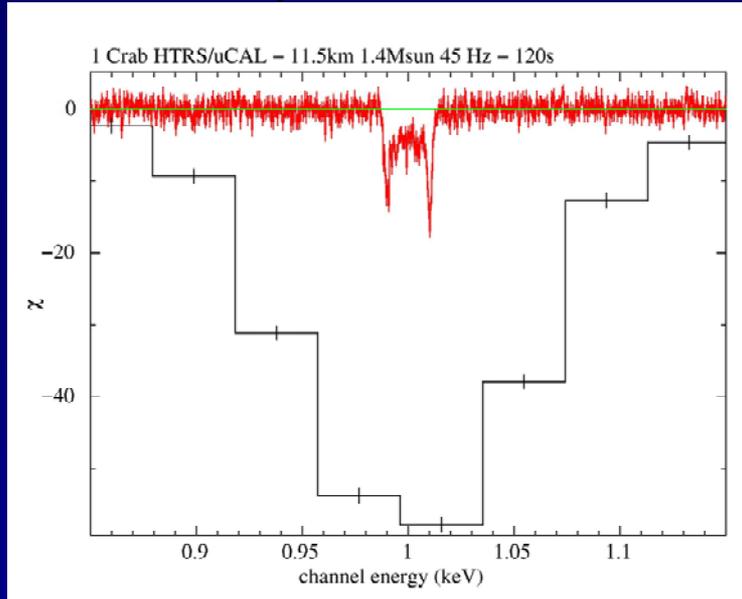
# Simulations

Spin = 45 Hz

Spin = 400 Hz

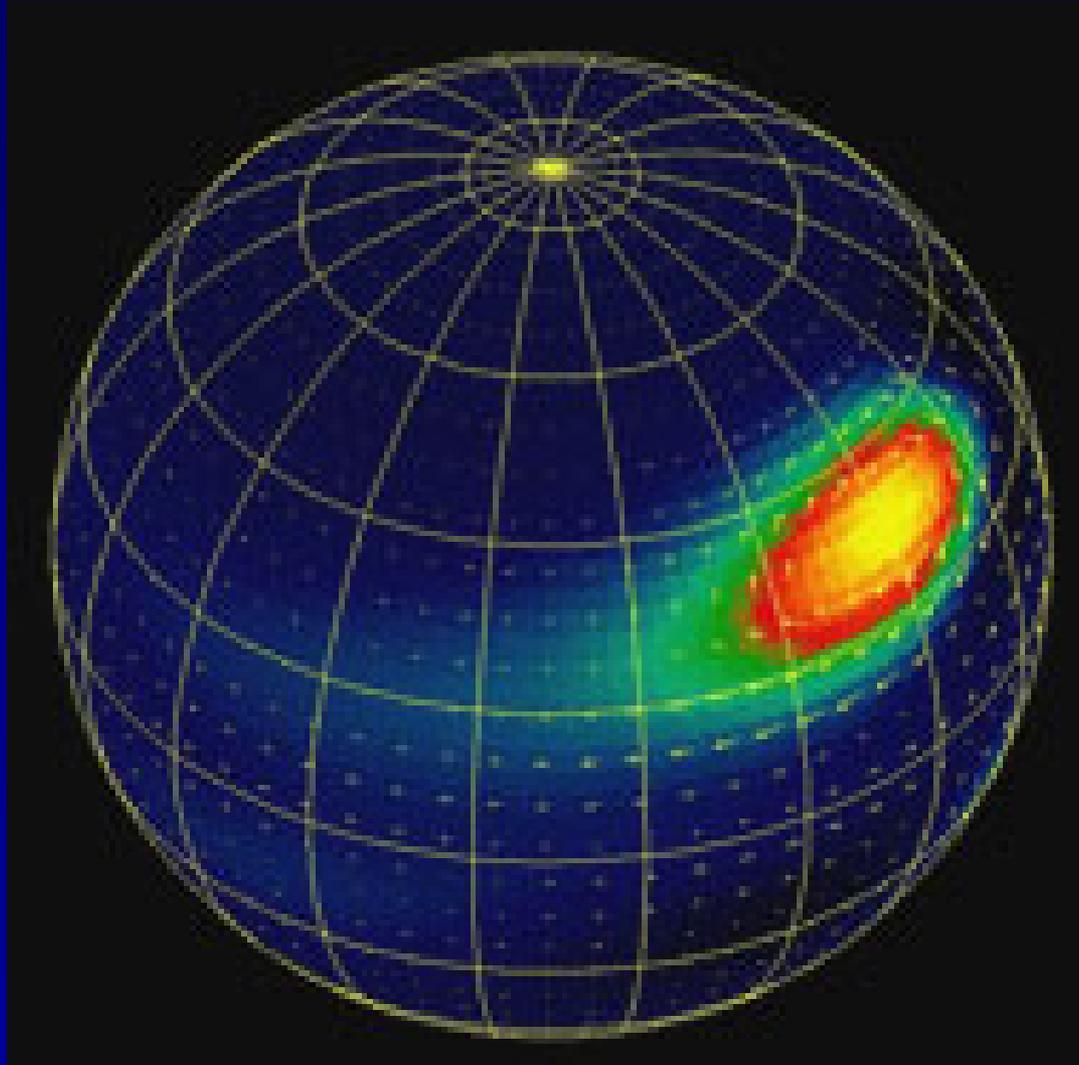
Fe XXVI  
 $z = 0.35$

$n = 2 - 3$   
Balmer  $\alpha$



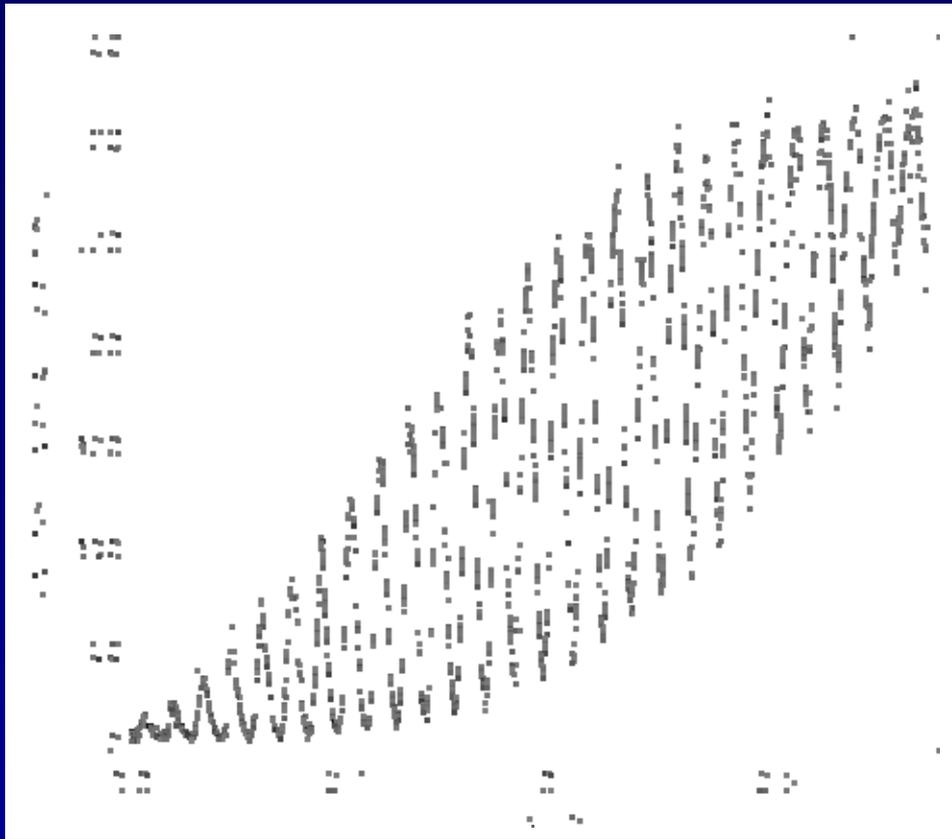
$n = 1 - 2$   
Lyman  $\alpha$

# Pulsations during X-ray bursts

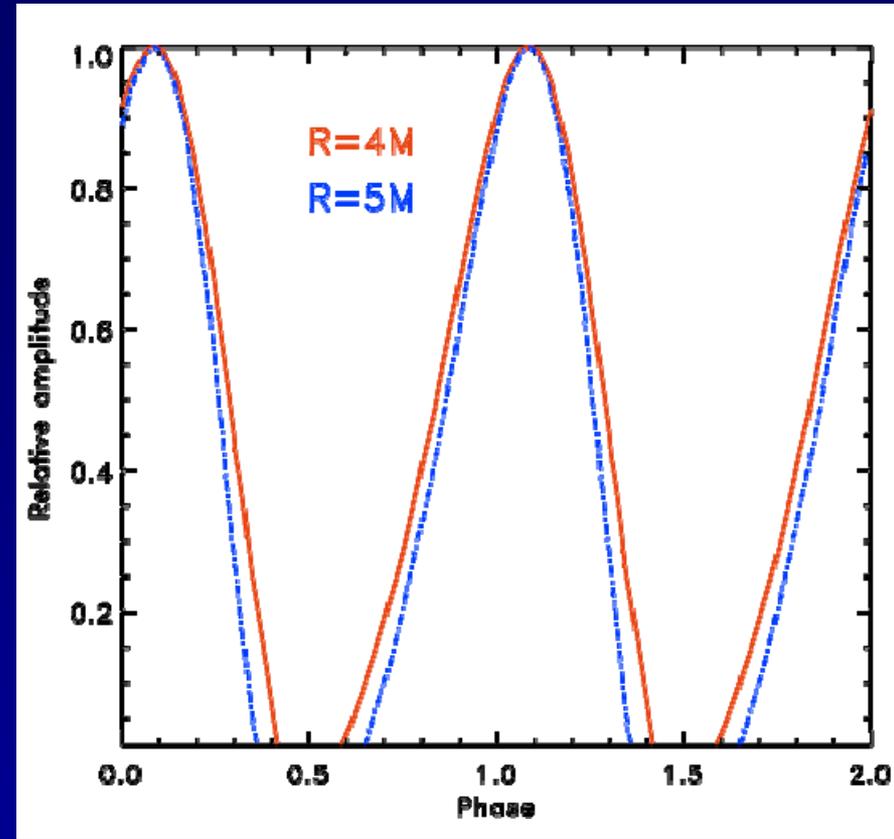


*Strohmayer et al.; Spitkovsky et al.*

# Pulsations during X-ray bursts

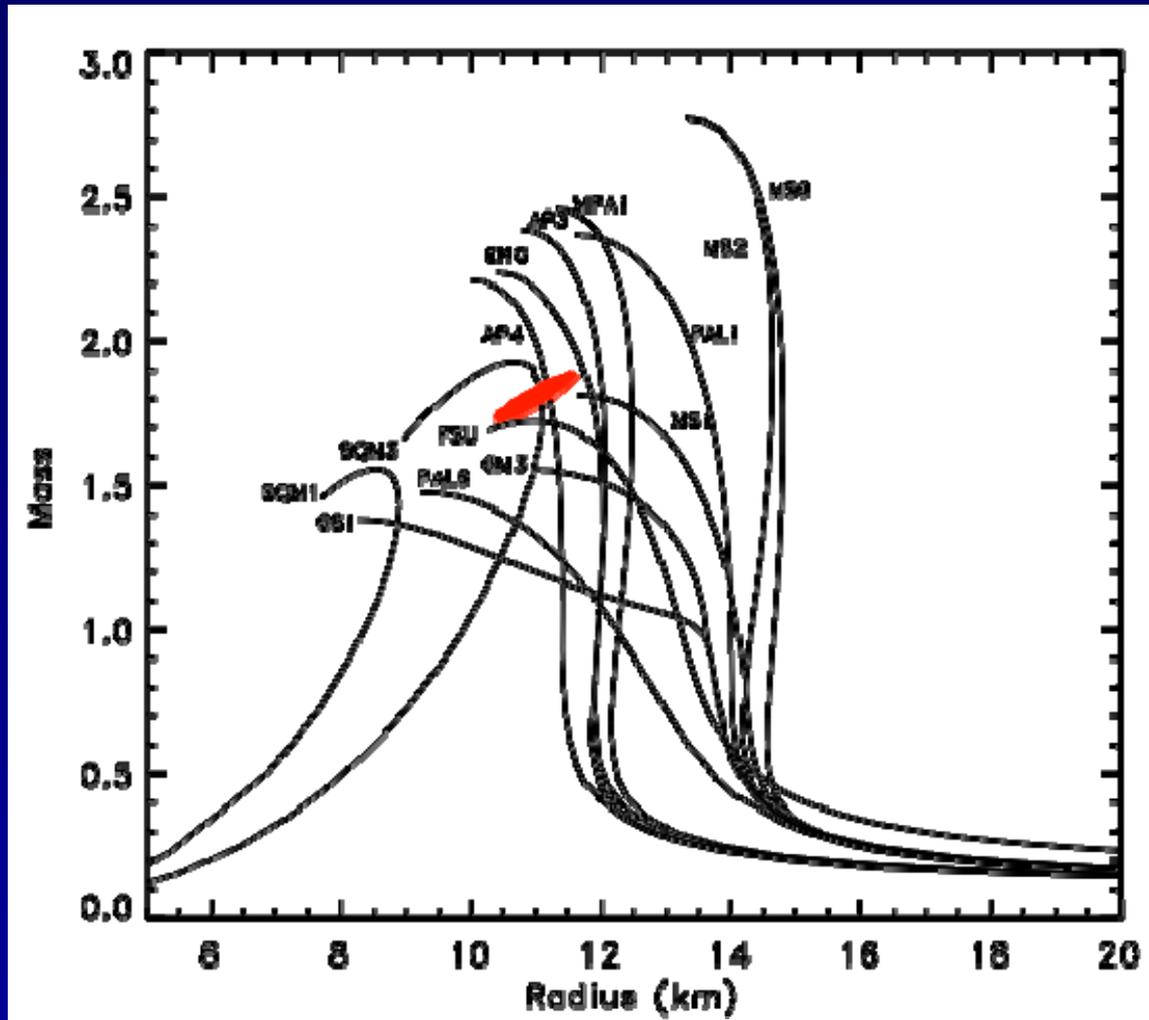


Simulated pulse profile for the rising phase of an X-ray burst (T. Strohmayer).



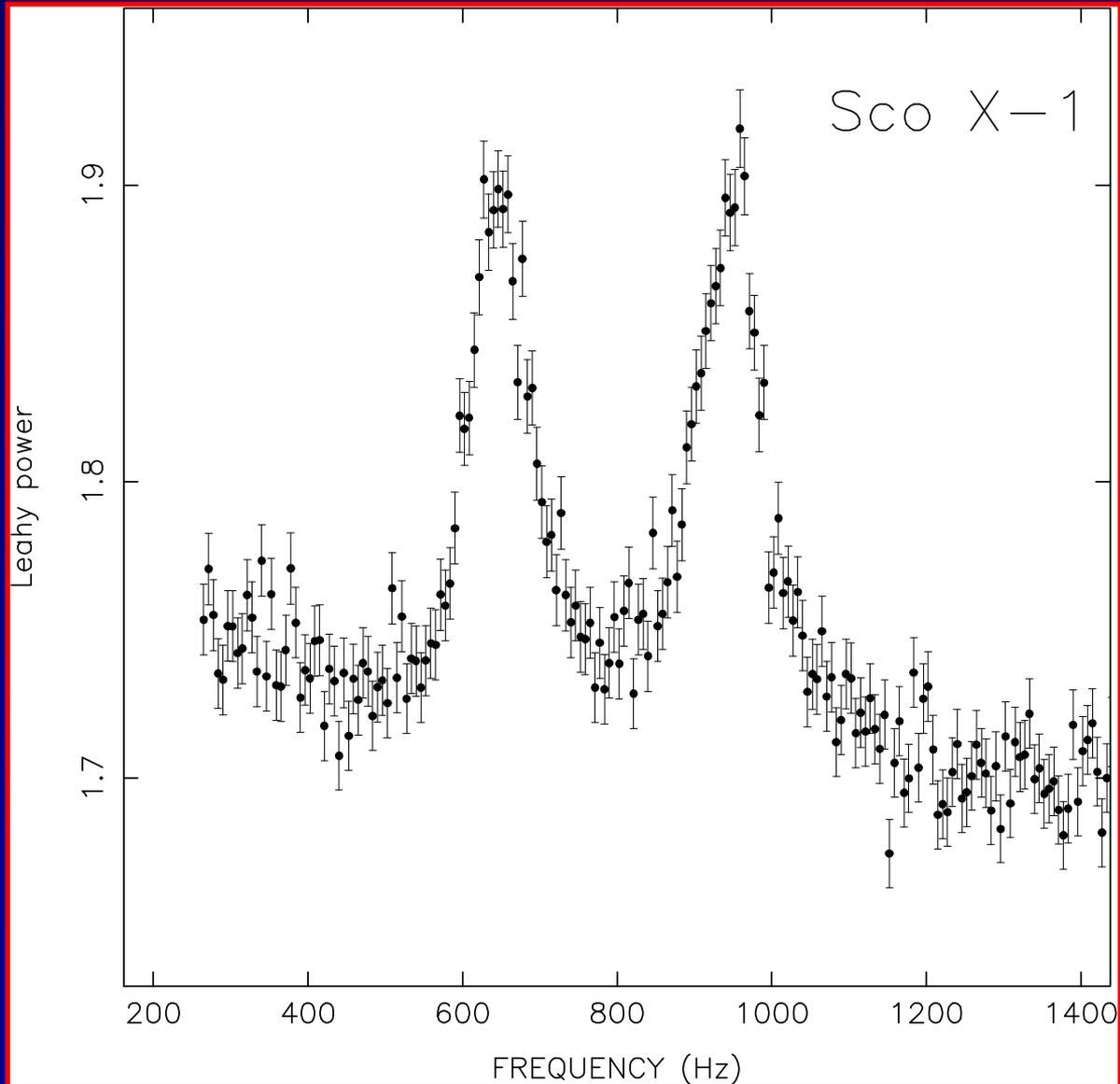
Pulse profiles for a 1.8 solar mass NS with a spin frequency of 364 Hz. (C. Miller).

# Pulsations during X-ray bursts

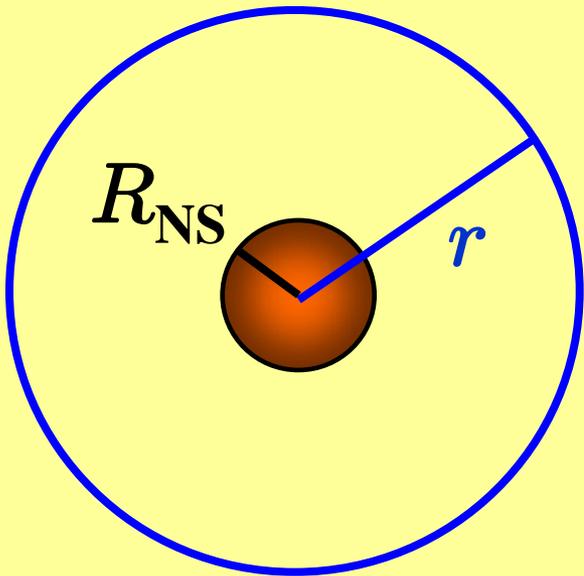


Mass and radius constraints from pulse-profile fitting. The red ellipse shows the 95% confidence regions from 5 typical bursts (C. Miller).

# Quasi-periodic oscillations



# Mass and radius constraints from timing



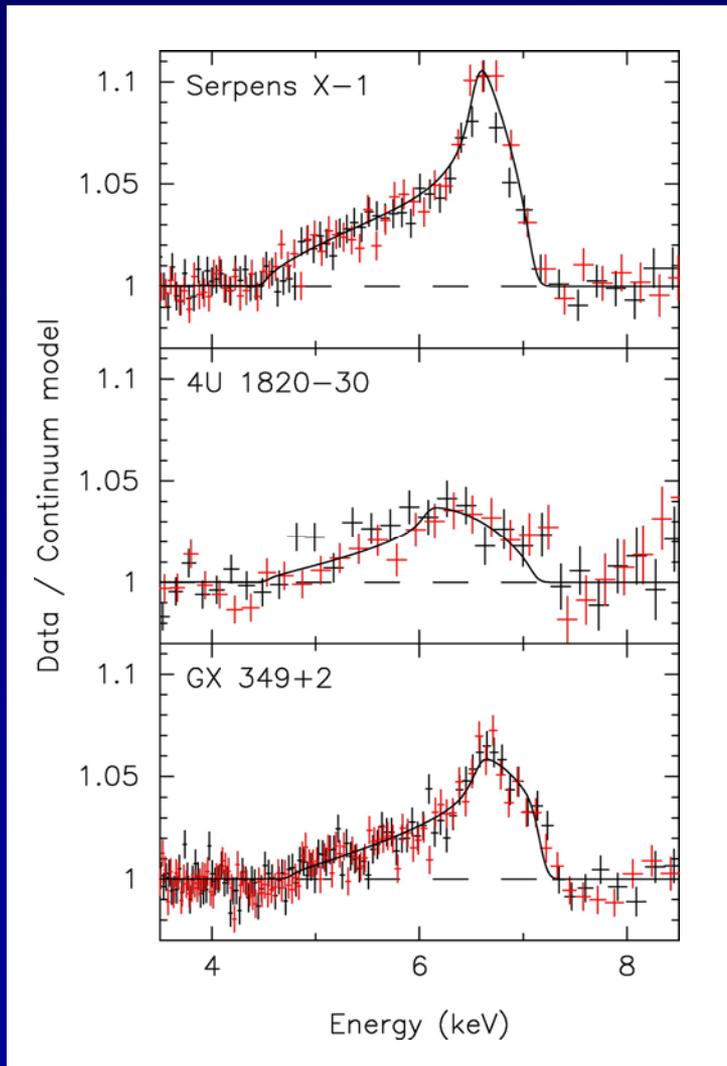
$$\nu = \frac{1}{2\pi} \sqrt{\frac{GM_{\text{NS}}}{r^3}}$$
$$R_{\text{NS}} \leq r$$

$$M_{\text{NS}} \leq 2.2(\nu/1000\text{Hz})^{-1} M_{\odot}$$

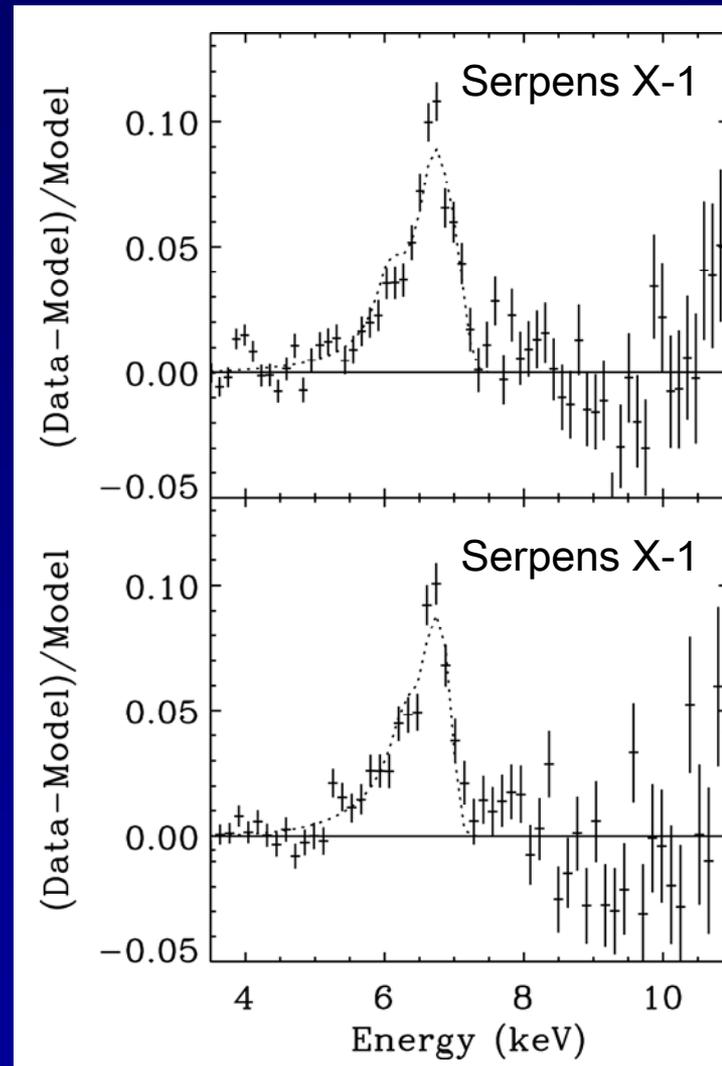
$$R_{\text{NS}} \leq 14.6(M_{\text{NS}}/M_{\odot})^{1/3}(\nu/1000\text{Hz})^{-2/3} \text{ km}$$

# Emission lines from the inner disc

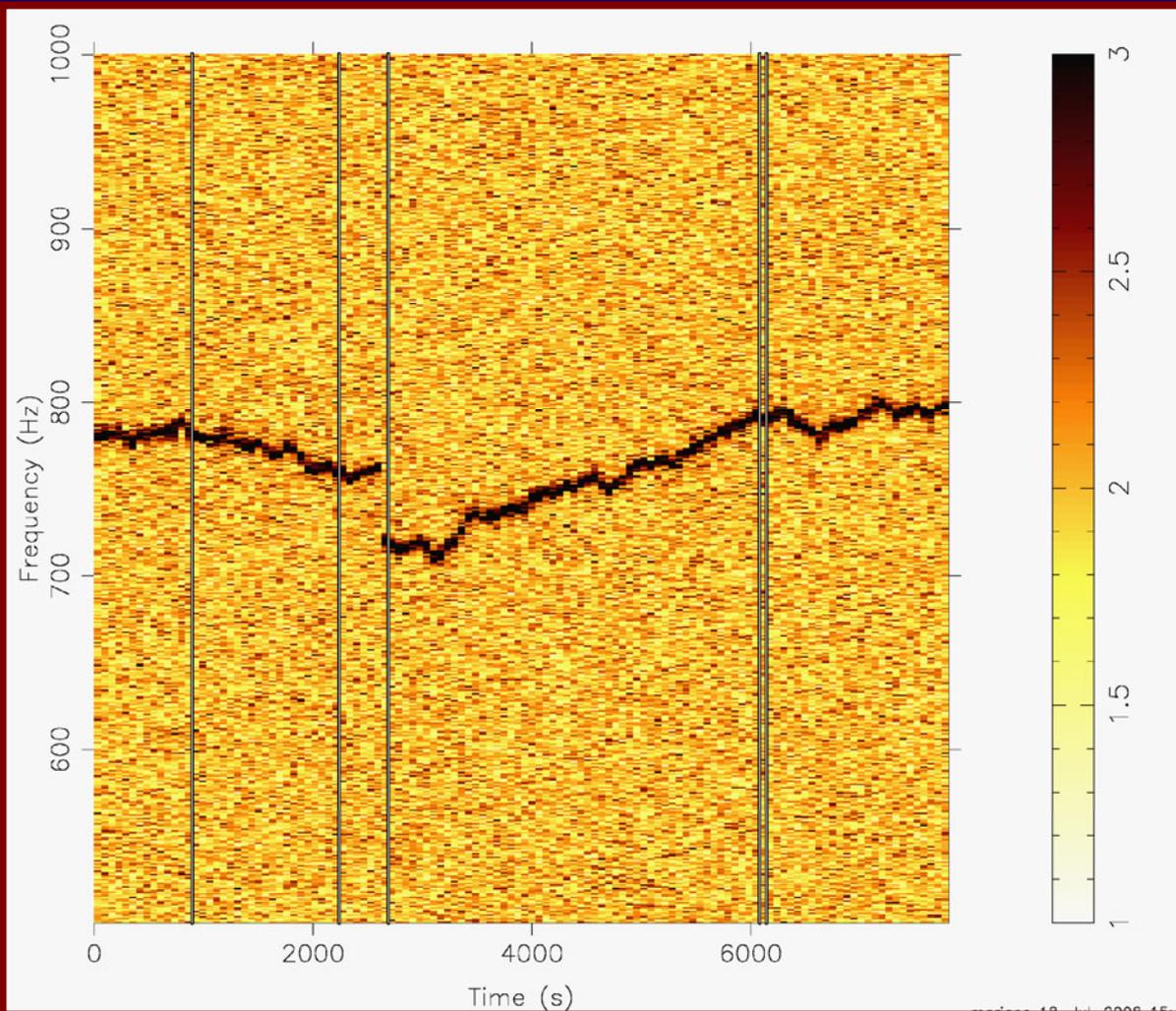
Suzaku



XMM-Newton



# Inner disc radius

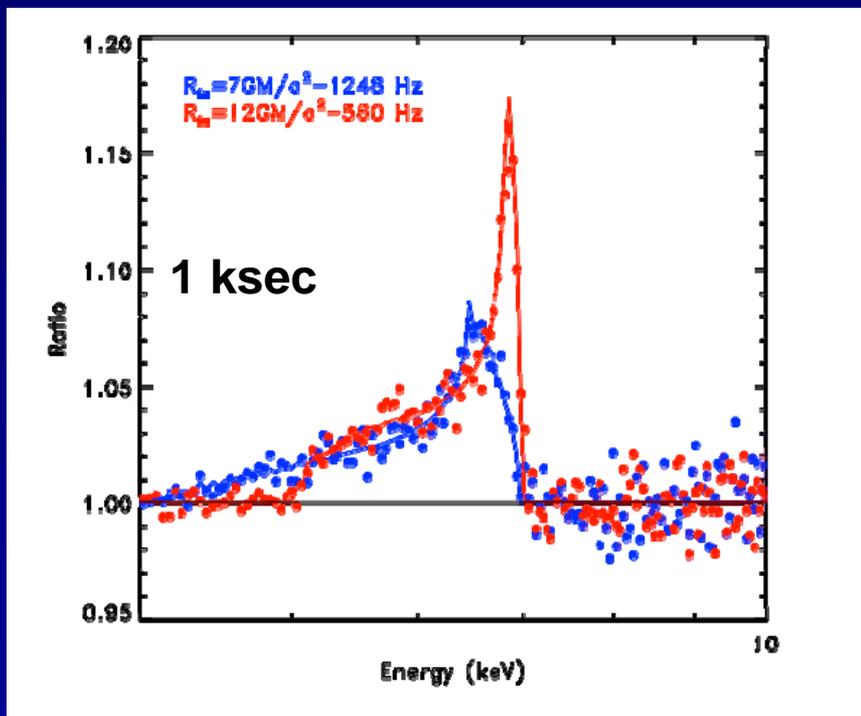


*Mendez et al.*

4U 1608-52

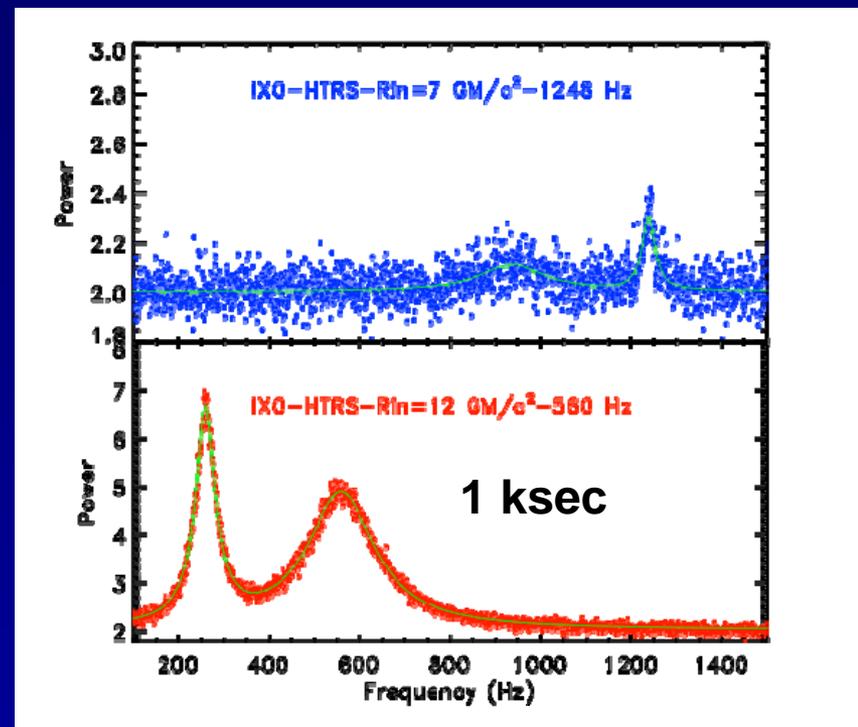
# Tracking the inner disc radius

Data / continuum



Energy (keV)

Power

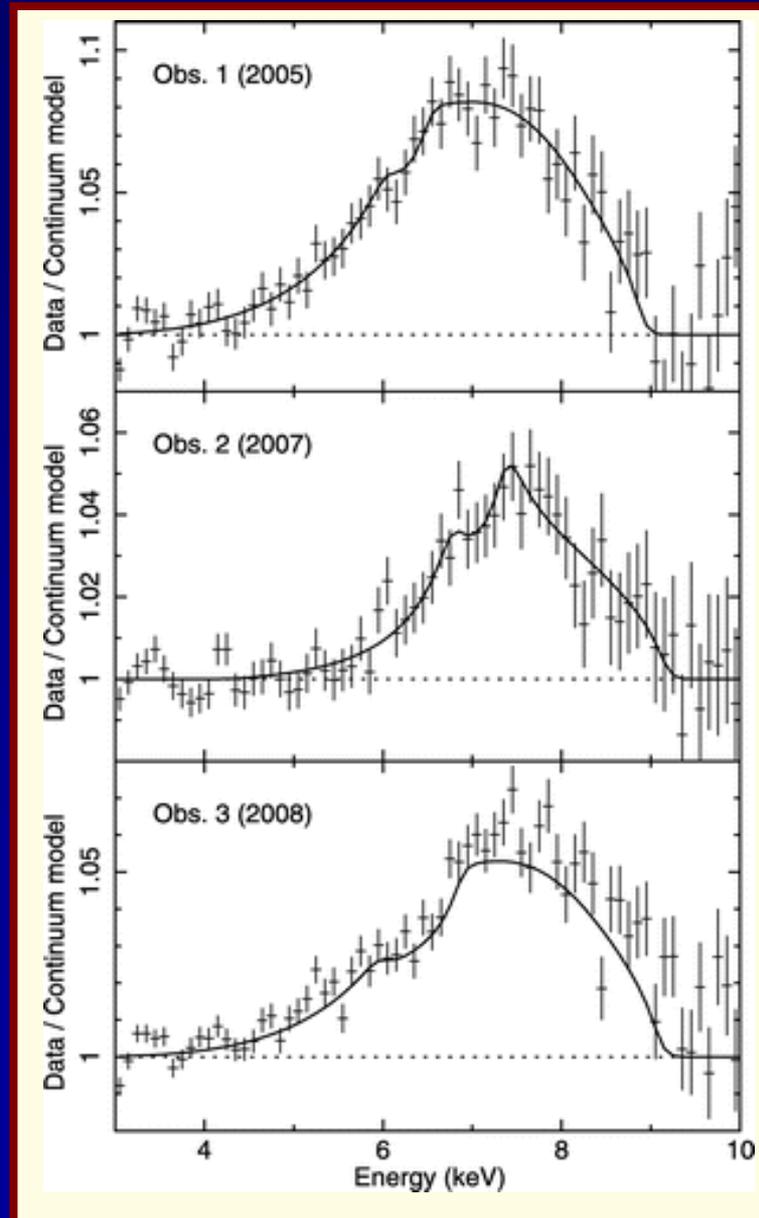


Frequency (Hz)

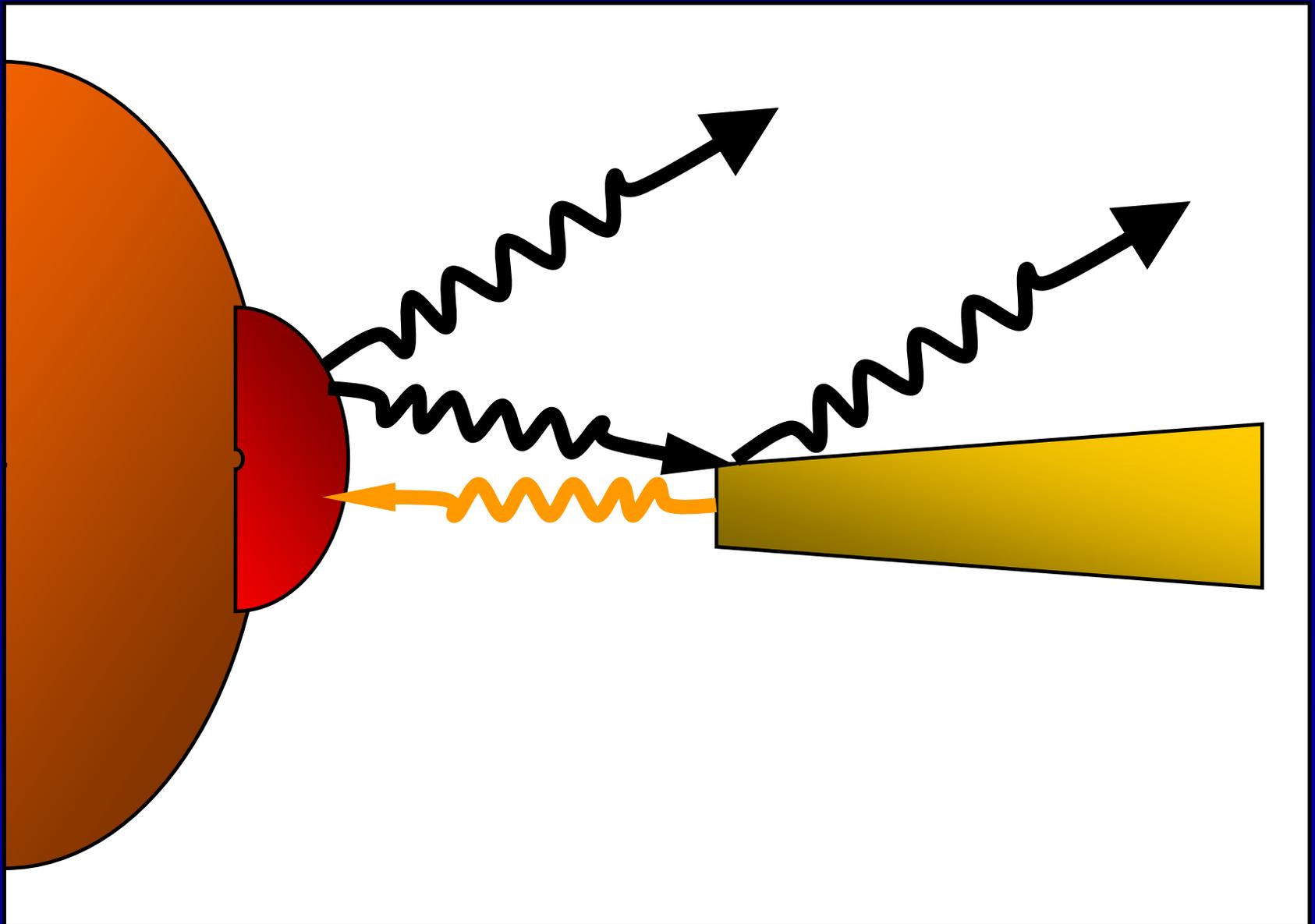
IXO/HTRS simulations by D. Barret

# Iron line

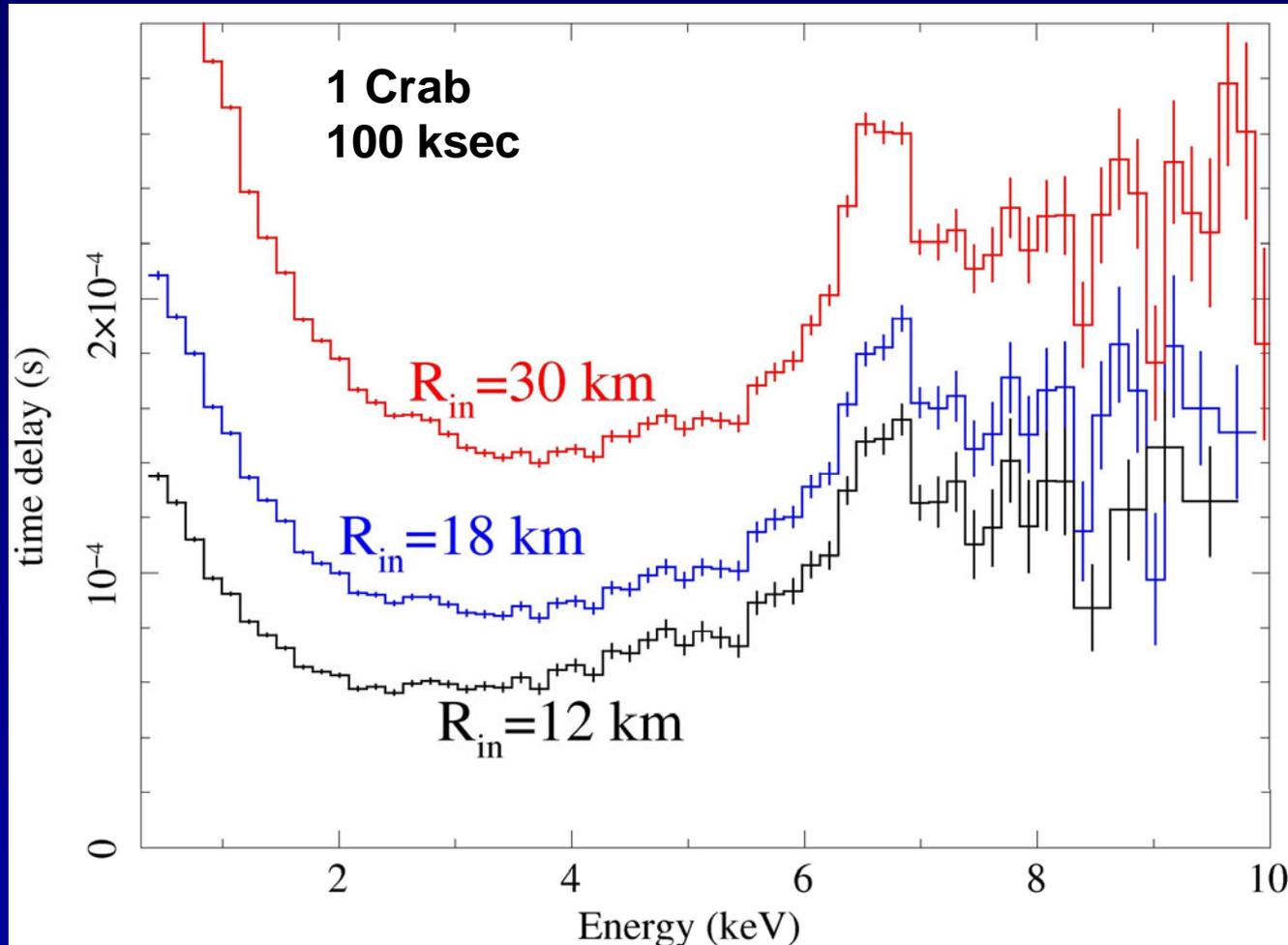
4U 1636-53



# Frequency-resolved time-delay spectrum



# Frequency-resolved time-delay spectrum



# Neutron-star EOS: Summary

- Multiple complementary (redundant) constraints of  $M$  and  $R$
- Requirements:
  - High time resolution (time scales  $\sim 1$  ms)
  - High count-rate capability (count rates  $\sim 1$  Crab or more)
  - Moderate spectral resolution (line broadening)
- HTRS is the primary instrument for this science
- Complementary information from XMS / WFI / XGS